

**Errata for "Fundamentals of Plasma Physics" by P. M. Bellan**  
**Hardcover edition, printed 2006**

**first errata version: June 19, 2006,**  
**last errata update: February 14, 2008**

Note: when possible, corrected items have a box around them to help identification.

Errata for each chapter starts on a separate page.

**Chapter 1**

**p. 30** (updated December 28, 2006). The top equation should read (in the published book, the right hand side was erroneously set in subscript font)

$$\mathbf{v}_{\text{new}} + \mathbf{A} \times \mathbf{v}_{\text{new}} = \mathbf{C}$$

## Chapter 2

**p. 49** (updated November 3, 2007 and February 14, 2008) All but the first sentence of the second paragraph in Sec. 2.5.2 should be deleted because if all the particles are put in one group, then the number of internal arrangements in this single group is  $f(1)! = N!$  and so  $C = N!/f(1)! = 1$  in which case the entropy is minimized (not maximized). However, the need to constrain maximization of entropy because of conservation of energy and particle number remains true and so the next paragraph is correct (the paragraph starting with "Thus, a qualification must be added to the argument. Randomizing...") and can be appended to the first sentence of the paragraph where the strikeout occurs.

After the strikeout in the second and the appending of the third paragraph, these combined two paragraphs should now read:

An important shortcoming of this argument is that it neglects any conservation relations that have to be satisfied. ~~To see this, note that the expression for entropy could be maximized if all the particles are put in one group, in which case  $C = N!$ , which is the largest possible value for  $C$ . Thus, the maximum entropy configuration of  $N$  plasma particles corresponds to all the particles having the same velocity. However, this would assign a specific energy to the system which would in general differ from the energy of the initial microstate. This maximum entropy state is therefore not accessible in isolated system, because energy would not be conserved if the system changed from its initial microstate to the maximum entropy state.~~ Thus, a qualification must be added to the argument. Randomizing...

### Chapter 3

**p.103** (updated December 28, 2006), Two lines after Eq.(3.112) should read:  
 “Evaluating Eq.(3.112) at  $s = \underline{\mathbb{S}\mathbb{A}}$ ,  $t = 0, \dots$ ”

**p. 128** (updated December 28, 2006), Eq.(3.185) should read

$$\begin{aligned}\frac{d\mathbf{v}_1}{dt} &= \frac{q}{m} [\mathbf{E}(\mathbf{x}_1, t) + \mathbf{v}_1 \times \mathbf{B}] \\ \frac{d\mathbf{v}_2}{dt} &= \frac{q}{m} [\mathbf{E}(\mathbf{x}_2, t) + \underline{\mathbf{v}\mathbb{2}} \times \mathbf{B}].\end{aligned}$$

**p.128**, Eq. (3.190) should be

$$\delta \ddot{y} + \omega_c^2 \left( 1 - \frac{\underline{\mathbb{1}}}{\omega_c^2} \frac{\partial \mathcal{E}_y}{\partial y} \right) \delta \dot{y} = \underline{\omega_c} \delta y \frac{\partial \mathcal{E}_x}{\partial y} \underline{\mathbb{1}} \delta y \frac{\partial}{\partial y} \left( \frac{d\mathcal{E}_y}{dt} \right). \quad (1)$$

**On pages 128-129** in the paragraph following Eq.3.190, the *five* places where the quantity  $\omega_c^{-1} \partial \mathcal{E}_y / \partial y$  appears should each be replaced by  $\underline{\omega_c^{-2}} \partial \mathcal{E}_y / \partial y$ .

**p.143** (updated December 28, 2006), problem 8, the sentence starting “Show by explicit evaluation..” should end with

“... and maximum axial field  $\underline{[(1 + \lambda)]} B_{\min}$  at  $z = L(t)$ .”

Also, the last sentence in problem 8 should end with:

“..... decreases below  $\theta_{trap} = \sin^{-1} \left( \underline{[1/\sqrt{1 + \lambda}]} \right)$ .”

## Chapter 4

**p. 158** (updated December 28, 2006), Equation 4.62a is missing a minus sign and should read

$$\frac{\partial \mathbf{B}_{\perp 1}}{\partial t} = \mp \frac{\partial}{\partial z} (\hat{z} \times \mathbf{E}_{\perp 1})$$

**p.160**, (updated November 3, 2007), Equation (4.70) should read:

$$\mathbf{J} \times \mathbf{B} = \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B} \mp \nabla \left( \frac{B^2}{2\mu_0} \right) + \frac{1}{\mu_0} \mathbf{B} \cdot \nabla \mathbf{B} .$$

## Chapter 5

**p. 188** (updated March 12, 2007) 3rd line above bottom of first paragraph had expression missing an "i" and should read

“Because  $f(\hat{p}) \rightarrow 0$  at the endpoints  $\beta \pm \infty, \dots$ ”

**p.191** (updated March 12, 2007) In paragraph beginning with “The problem can be simplified...”, there was an extra ‘i’ in the second sentence which should read

“Because each term in Eq.(5.66) has a factor  $\exp(p_j t), \dots$ ”

**p.194** (updated March 12, 2007)

Equation (5.78) was missing some  $\eta$  factors and a P for principle part and should read

$$\begin{aligned}
 P \frac{1}{\pi^{1/2}} \int_{-\infty}^{\infty} d\xi \frac{\exp(-\xi^2)}{(\xi - \alpha)} &= \frac{1}{\pi^{1/2}} P \int_{-\infty}^{\infty} d\eta \frac{e^{-\eta^2 - 2\alpha\eta - \alpha^2}}{\eta} \\
 &= \frac{e^{-\alpha^2}}{\pi^{1/2}} P \int_{-\infty}^{\infty} d\eta \frac{e^{-\eta^2}}{\eta} \left[ 1 - 2\alpha\eta + \frac{(-2\alpha\eta)^2}{2!} \right. \\
 &\quad \left. + \frac{(-2\alpha\eta)^3}{3!} + \dots \right] \\
 &= -2\alpha \frac{e^{-\alpha^2}}{\pi^{1/2}} \int_{-\infty}^{\infty} d\eta e^{-\eta^2} \left[ 1 + \frac{2\eta^2\alpha^2}{3} + \dots \right] \\
 &= -2\alpha (1 - \alpha^2 + \dots) \left( 1 + \frac{\alpha^2}{3} + \dots \right) \\
 &= -2\alpha \left( 1 - \frac{2\alpha^2}{3} + \dots \right)
 \end{aligned} \tag{2}$$

## Chapter 6

p.209, Equation 6.11 should read (updated Feb. 14, 2008)

$$\begin{aligned}\overleftrightarrow{\mathbf{K}} \cdot \tilde{\mathbf{E}} &= \tilde{\mathbf{E}} - \boxed{\sum_{\sigma=i,e}} \frac{\omega_{p\sigma}^2}{\omega^2} \left[ \tilde{E}_z \hat{z} + \frac{\tilde{\mathbf{E}}_{\perp}}{1 - \omega_{c\sigma}^2/\omega^2} - \frac{i\omega_{c\sigma}}{\omega} \frac{\hat{z} \times \tilde{\mathbf{E}}}{1 - \omega_{c\sigma}^2/\omega^2} \right] \\ &= \begin{bmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{bmatrix} \cdot \tilde{\mathbf{E}}\end{aligned}\tag{3}$$

## Chapter 8

**p. 277**, 6th line from top missed a factor of  $k_{\parallel}^2$  and should be:

"using  $1/2\lambda_{D\sigma}^2\alpha_0^2 = \boxed{k_{\parallel}^2}\omega_{p\sigma}^2/\omega^2$ , Eq. (8.36) becomes"

**p. 277**, Eq.(8.47) was also missing a factor of  $k_{\parallel}^2$  in top of RHS and should be

$$k_{\parallel}^2 + k_{\perp}^2 + \sum_{\sigma} (1 - k_{\perp}^2 r_L^2) \left( \begin{array}{l} \left( 1 + \frac{k_{\perp}^4 r_L^4}{4} \right) \left( -\frac{\omega_{p\sigma}^2 \boxed{k_{\parallel}^2}}{\omega^2} + i\frac{\alpha_0 \sqrt{\pi}}{\lambda_{D\sigma}^2} e^{-\alpha_0^2} \right) \\ + \frac{k_{\perp}^2 \omega_{p\sigma}^2}{2\omega_{c\sigma}^2} \left[ \begin{array}{l} -\frac{2\omega_{c\sigma}^2}{\omega^2 - \omega_{c\sigma}^2} \\ + i\alpha_0 \sqrt{\pi} \left( \begin{array}{l} e^{-\alpha_1^2} \\ + e^{-\alpha_{-1}^2} \end{array} \right) \end{array} \right] \\ + \frac{k_{\perp}^4 r_L^2 \omega_{p\sigma}^2}{8\omega_{c\sigma}^2} \left[ \begin{array}{l} -\frac{8\omega_{c\sigma}^2}{\omega^2 - 4\omega_{c\sigma}^2} \\ + i\alpha_0 \sqrt{\pi} \left( \begin{array}{l} e^{-\alpha_2^2} \\ + e^{-\alpha_{-2}^2} \end{array} \right) \end{array} \right] \end{array} \right) = 0.$$

**p.283** The fifth line of Eq.(8.71) was missing a factor of 2 inside the square root and should be:

$$\begin{aligned} y(x) &\simeq \int_{-\infty}^{\infty} e^{f(p_s) - r^2 |f''(p_s)|/2} dr e^{i\theta} \\ &= e^{f(p_s) + i\theta} \int_{-\infty}^{\infty} e^{-r^2 |f''(p_s)|/2} dr \\ &= e^{f(p_s) + i\theta} \sqrt{\frac{2\pi}{|f''(p_s)|}} \\ &= e^{f(p_s)} \sqrt{\frac{2\pi e^{i2\theta}}{|f''(p_s)|}} \\ &= e^{f(p_s)} \sqrt{\frac{\boxed{2}\pi e^{i(\pm\pi - \psi)}}{|f''(p_s)|}} \\ &= e^{f(p_s)} \sqrt{-\frac{2\pi}{f''(p_s)}}. \end{aligned}$$

**p. 288**, Line 6 of list item 4 (i.e. item at top of page) should read:  
 “allow the upper sign  $\overline{\text{plus}}$  for the small root (i.e. cold mode)...”

**p.288**, Equation (8.95) should have the factor  $i$  outside of the parenthesis in the upper term on the RHS and so should read

$$\left\{ \begin{array}{l} i \frac{\exp(-\frac{2}{3}|\xi|^{3/2})}{|\xi|^{3/4}} \\ + \frac{\exp[-2\mu|\xi|^{1/2}]}{(|\xi|\mu)^{1/4}} \end{array} \right\} \iff \left\{ \begin{array}{l} \frac{\exp[\frac{2}{3}\overline{\text{plus}}(\xi)^{3/2}]}{i^{1/2}\xi^{3/4}} \\ + i^{1/2} \frac{\exp[2i(\mu\xi)^{1/2}]}{(\xi\mu)^{1/4}} \end{array} \right\}$$

evanescent side,  $\xi < 0$

propagating side,  $\xi > 0$



## Chapter 9

**p.320**, Fourth line after Eq.(9.33) should read:

” Thus the right hand side term scales as  $\int ds B^2 r \sim r^{-3}$  and so vanishes as  $r \rightarrow \infty$ . This is in contradiction to the left hand side being positive definite and so the set of initial assumptions must be erroneous.

**p.330**, (updated November 3,2007)

phrase immediately after Eq.(9.73) should read:

showing that  $I$  acts like a stream-function for the current.

**p. 339**, Problem 3. In order to be dimensionally correct, the flux given on the second line should be:

$$\psi = B_0 r^2 (2a^2 - r^2 - 4(\alpha z)^2) / a^2$$

**p.340**, (updated March 12, 2007) Problem 5. Lawson criterion: the neutron and alpha particle energies were interchanged. The third sentence should read

“The output energy consists of 14.1 MeV neutron kinetic energy and 3.5 MeV alpha particle kinetic energy.”

## Chapter 10

**p. 349**, third line from the bottom should not have a  $\Delta y$  factor in the expression for  $\gamma^2$  and so read:  
 "unstable stratum giving a growth rate  $\gamma^2 \sim g\rho_0^{-1} \partial\rho_0/\partial y$  where  $\partial\rho_0/\partial y$  is the value in the unstable region"

**p. 350**, second line from top should not have a  $\Delta y$  factor in the expression for  $\gamma^2$  and was missing a  $\rho_0$  in the denominator of the second term on the RHS and so should read:  
 "reducing the growth rate to  $\gamma^2 \sim g\rho_0^{-1} \partial\rho_0/\partial y - (\mathbf{k} \cdot \mathbf{B}_0)^2 / \mu_0 \rho_0$ "

**p.358**, third line after Eq.(10.79) had a period instead of a dot product at the end of the line and the end of the line should be

".... since  $\int d^3r (\boldsymbol{\xi}_r + i\boldsymbol{\xi}_i) \boxtimes$ "

**p.381**, Eq.(10.180) is missing a term 1/2 and should read:

$$\frac{1}{2} + \bar{B}_{0vz}^2 [1 - k^2 a^2 \ln(|k|a)] > \bar{P}_0 \implies \text{stable.}$$

**p.384** (updated November 3,2007), problem 6(b) should read:  
 Suppose  $\psi(x, z) = f(z) \sin(kx)$ .

**p.384**, problem 6(d) should read:  
 "Sketch the projection in the  $z = 0$  plane of the field line starting from  $x = -\pi/2k$  to  $x = +\pi/2k$  in the  $z = 0$  plane. Do this for a sequence of increasing values of  $\lambda^2$ . What happens to the projection of the field line as  $\lambda^2$  is increased? How is current related to  $\lambda$ ?

## Chapter 11

p. 407, bottom line in page should read:  
"as  $\mathbf{E} = -\nabla\varphi - \delta\mathbf{A}/\delta t$ ....."

## Chapter 13

p. 446 Equation 13.58 should read

$$n_i Z = n_e$$

## Chapter 14

**p. 489**, (updated November 3, 2007) Assignment 1(a), 2nd line should read:

"in Assignment 2 of Chapter 13...."

**p. 490**, middle unnumbered equation had wrong power of  $k$  and should read

$$\mathcal{E}(\omega_r/v, t) \sim k^2 |\phi|^2$$

and similarly the bottom unnumbered equation should read

$$\bar{D}_{QL}(w) \sim \frac{8\pi^2 k^2 |\phi|^2}{m_e w \omega_{pe}^2 \ln \Lambda}$$

## Chapter 15

**p. 502**, Eq. (15.44) has the wrong sign on the RHS and should be:

$$\nabla \times \tilde{\mathbf{B}} - \mu_0 \varepsilon_0 \frac{\partial \tilde{\mathbf{E}}}{\partial t} - \mu_0 \sum_{\sigma} n_{\sigma} q_{\sigma} \tilde{\mathbf{u}}_{\sigma} = \mu_0 \sum_{\sigma} \tilde{n}_{\sigma} q_{\sigma} \tilde{\mathbf{u}}_{\sigma}$$

**p. 503** (updated November 3, 2007) Eq.15.50 had  $k_3^3$  instead of  $k_3^2$  and should read

$$\frac{\omega_3^2}{k_3^2} > \frac{\omega_2^2}{k_2^2} \quad \text{and} \quad \frac{\omega_3^2}{k_3^2} > \frac{\omega_1^2}{k_1^2}$$

**p. 527**, Problem 1(c), had wrong subscripts and should read:

"Use Eqs.(15.24) to write  $A_1^2(t)$  and  $A_2^2(t)$  in terms of  $A_3^2(t)$  and the initial conditions  $A_1^2(0)$ ,  $A_2^2(0)$ , and  $A_3^2(0)$ ."

**p. 527**, Problem 1(e), had wrong subscripts in the second sentence and should read:

" What is the total energy of this pseudo-particle in terms of  $A_1^2(0)$ ,  $A_2^2(0)$ , and  $A_3^2(0)$ ?"

## Appendix A

p. 585, The first two lines in the "Summary of vector identities" should be:

$$\begin{aligned}\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) &= \mathbf{B} (\mathbf{A} \cdot \mathbf{C}) - \mathbf{C} (\mathbf{A} \cdot \mathbf{B}) \\ (\mathbf{A} \times \mathbf{B}) \times \mathbf{C} &= \mathbf{B} (\mathbf{A} \cdot \mathbf{C}) - \mathbf{A} (\mathbf{B} \cdot \mathbf{C})\end{aligned}$$

Derivation of one of the standard vector identities was missing from this Appendix and should be:

The expression  $\nabla \times (\mathbf{B} \times \mathbf{C})$  can be evaluated using the vector cross-product rule in conjunction with the product rule. Thus, the rule  $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B} (\mathbf{A} \cdot \mathbf{C}) - \mathbf{C} (\mathbf{A} \cdot \mathbf{B})$  imposed by Eq.(A.1) must be satisfied with the  $\nabla$  operator playing the role of  $\mathbf{A}$ , and also the product rule requirement  $(\psi\chi)' = \chi'\psi + \chi\psi'$  must be satisfied. Since  $\mathbf{B} (\mathbf{A} \cdot \mathbf{C}) = (\mathbf{C} \cdot \mathbf{A}) \mathbf{B}$  we must have  $\nabla$  operate on both  $\mathbf{B}$  and on  $\mathbf{C}$  as a derivative operator and in addition satisfy the vector requirement. Similarly, we can use  $\mathbf{C} (\mathbf{A} \cdot \mathbf{B}) = (\mathbf{B} \cdot \mathbf{A}) \mathbf{C}$  in order to give  $\nabla$  represented by  $\mathbf{A}$  a chance to operate on both  $\mathbf{B}$  and on  $\mathbf{A}$  according to the product rule. Thus, using the vector rule  $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B} (\mathbf{A} \cdot \mathbf{C}) - \mathbf{C} (\mathbf{A} \cdot \mathbf{B})$ , letting  $\nabla$  replace  $\mathbf{A}$ , and taking into account the conditions imposed by the product rule of calculus gives

$$\nabla \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B} (\nabla \cdot \mathbf{C}) + \mathbf{C} \cdot \nabla \mathbf{B} - \mathbf{C} (\nabla \cdot \mathbf{B}) - \mathbf{B} \cdot \nabla \mathbf{C}.$$

The above expression should be added to the "Summary of vector identities".

## Appendix B

**p.592** (updated November 3, 2007)

The third line in the curl identity had a derivative with respect to  $\phi$  instead of with respect to  $\theta$  and should read

$$\begin{aligned}\nabla \times \mathbf{V} &= \frac{1}{r \sin \theta} \left( \frac{\partial}{\partial \theta} (V_\phi \sin \theta) - \frac{\partial V_\theta}{\partial \phi} \right) \hat{r} \\ &+ \frac{1}{r \sin \theta} \left( \frac{\partial V_r}{\partial \phi} - \frac{\partial}{\partial r} (V_\phi r \sin \theta) \right) \hat{\theta} \\ &+ \frac{1}{r} \left( \frac{\partial}{\partial r} (V_\theta r) - \frac{\partial V_r}{\partial \theta} \right) \hat{\phi}\end{aligned}$$